Module 3e: Comparison of Pipe Flow Equations and Head Losses in Fittings

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Comparison of Three Head Loss Equations

Using flow and SI units:

- Darcy-Weisbach:

$$h_f = \frac{8fLQ^2}{\pi^2 g D^5}$$

$$\frac{h_f}{L} = S = \frac{8fQ^2}{\pi^2 g D^5}$$

$$S = \left(\frac{8f}{\pi^2 g}\right)\frac{Q^2}{D^5}$$

Comparison of Three Head Loss Equations

Using flow and SI units:

- Hazen-Williams:

$$Q = 0.278CD^{2.63}S^{0.54}$$

$$S^{0.54} = \frac{Q}{0.278CD^{2.63}}$$

$$S = \left(\frac{Q}{0.278CD^{2.63}}\right)^{1/0.54}$$

$$S = \left(\frac{Q}{0.278CD^{2.63}}\right)^{1.85}$$

$$S = \frac{10.67Q^{1.85}}{C^{1.85}D^{4.87}}$$

$$S = \left(\frac{10.67}{C^{1.85}}\right)\frac{Q^{1.85}}{D^{4.87}}$$

Comparison of Three Head Loss Equations

Using flow and SI units:

- Manning's:

$$Q = \frac{0.312}{n}D^{8/3}S^{1/2}$$

$$S^{1/2} = \frac{Qn}{0.312D^{8/3}}$$

$$S = \left(10.27n^2\right)\frac{Q^2}{D^{16/3}}$$

$$S = (10.27n^2)\frac{Q^2}{D^{5.33}}$$

Comparison of Three Head Loss Equations

- Note that all of the equations are in approximately the same form:

$$S = K\frac{Q^2}{D^5}$$

- Setting all the slopes equal to each other:

$$S = \left(\frac{8f}{\pi^2 g}\right)\frac{Q^2}{D^5} = 10.27n^2\frac{Q^2}{D^{5.33}} = \left(\frac{10.67}{C^{1.85}}\right)\frac{Q^{1.85}}{D^{4.87}}$$

Comparison of Three Head Loss Equations

- Dividing through by Q^2/D^5 and substituting:

$$0.0827f = 10.27n^2\left(\frac{1}{D^{0.33}}\right) = \left(\frac{10.67}{C^{1.85}}\right)\frac{Q^{-0.15}}{D^{-0.13}}$$

$$0.0827f = \frac{10.3n^2}{D^{1/3}} = \frac{10.7D^{0.13}}{C^{1.85}Q^{0.15}}$$

Most commonly used water supply pipes

- Iron Pipe
- Steel Pipe
- PVC Pipe
- Concrete Pipe
- *Asbestos-cement Pipe*
- *Clay Pipe*
- *Lead Pipe*

Iron Pipe

- – Most widely used material
- – Safe to assume 100 year service life
- – Subject to corrosion and tuberculation
- – Pipes are often coated to protect against corrosion
  - Tar coating used inside and out
  - Cement lined (BWWB)
  - Bituminous enamel inside and out

Steel Pipe

- Frequently used for pipe lines, trunk mains where pressures are high
- Seldom used for distribution system because harder to make good connections
- Cheaper than iron
- Can't withstand external loads as well as iron
- Average life 25 – 50 years
- Even more important to protect from corrosion (plastic coatings, cement, tar)

PVC Pipe

- Widely used in domestic plumbing and water distribution systems
- Easier to handle
- Cheaper than iron or steel
- May have short life (25 years)

Concrete Pipe

- Reinforced if >24" diameter
- Not subject to tuberculation
- Life about 75 years

Asbestos-cement Pipe

- Not installed much any more due to asbestos exposure during installation
- Portland cement with asbestos reinforcing fibers
- Immune to soil corrosion
- Much in use world wide
- Potential asbestos fiber problems in drinking water?

Vitrified Clay

- Seldom used in U.S. today
- Kiln fired
- Sodium chloride added to form hard waterproof glaze
- Not subject to chemical or bacterial corrosion
- Sizes 4 – 42 inches

Pipe Sizes in Metric and U.S. Customary Units

Metric Sizes, mm	Existing U.S. Pipe Sizes (in.)	Actual Sizes (mm)
100	4	101.6
125	5	127.0
150	6	152.4
200	8	203.2
250	10	254.0
300	12	304.8
350	14	355.6
375	15	381.0
400	16	406.4
450	18	457.2

Pipe Sizes in Metric and U.S. Customary Units

Metric Sizes, mm	Existing U.S. Pipe Sizes (in.)	Actual Sizes (mm)
500	20	508.0
525	21	533.4
600	24	609.6
675	27	685.8
750	30	762.0
900	36	914.4
1050	42	1066

From: Metcalf and Eddy, Inc. and George Tchobanoglous. *Wastewater Engineering: Collection and Pumping of Wastewater*. McGraw-Hill, Inc. 1981.

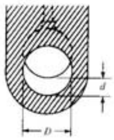
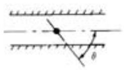
Head Loss in Fittings

- Losses in fittings vary with the velocity head. The general form of the head loss equation is:

$$h_m = K\frac{V^2}{2g}$$

Where $h_m = h_L$ = head loss through fitting ('m' for minor loss)
K (or C) = loss coefficient
V = velocity through fitting
g = gravitational constant

Head Loss in Fittings

Cause of minor loss	K value or loss expression	Reference
Gate valve Fig. C-1 	see table below. For larger sizes, values for 300-mm valve may be used.	2
Butterfly valve Fig. C-2 	see table below. Values of K depend on size of valve and design of disk. At wide-open position, K may vary from 0.3 to 1.3. Values of K at part-open positions vary widely. Consult manufacturer's catalog for values for a specific valve. The K values given are representative.	3, 5

Gate valve, d/D:

D, mm	in	1/8	1/4	3/8	1/2	3/4	1
50	2	140	20	6.5	3.0	0.68	0.16
100	4	91	16	5.6	2.6	0.55	0.14
150	6	74	14	5.3	2.4	0.49	0.12
200	8	66	13	5.2	2.3	0.47	0.10
300	12	56	12	5.1	2.2	0.47	0.07

Butterfly valve:

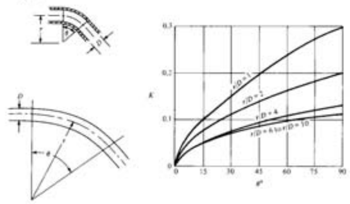
φ	5°	10°	20°	30°	40°	50°	60°	70°
K	0.24	0.52	1.54	3.91	10.8	32.6	118	751

Head Loss in Fittings

Cause of minor loss	K value or loss expression	Reference
Swing check valve Fig. C-3	2.5 (fully open)	1, 4, 7, 8
90° elbow Fig. C-4 Regular	0.3	4, 9
Fig. C-5 Long radius	0.2	4, 9
45° elbow Fig. C-6	0.2	4, 9
Return bends (θ = 180°) Fig. C-7	0.4	

Head Loss in Fittings

Cause of minor loss	K value or loss expression	Reference
Bends Fig. C-8	see chart	
Miter bends Fig. C-9	$1.5(1 - \cos\theta)$	3, 6
Tee Fig. C-10 Branch flow	AWWA STD[a]: 0.5 | Zero radius[b]: 1.0	
Fig. C-11	0.55 | 1.00	

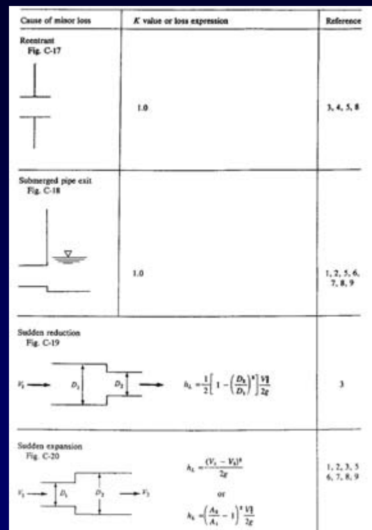


Head Loss in Fittings

Cause of minor loss	K value or loss expression	Reference
Fig. C-12 Line flow	0.1 | 0	
Fig. C-13	0.5 | 0.75	
Fig. C-14	A 0.5 | 1.0; B 0.05 | 0.05. [a]Use for DIP, RCP, etc. [b]Use for welded steel pipe	
Pipe entrance Flush Fig. C-15	0.5	1, 2, 3, 4,
Rounded Fig. C-16	see table below	3, 8, 2

Description	K
$r/D \geq 0.18$	0.05
Well rounded	0.01–0.05
Rounded	0.04–0.2

Head Loss in Fittings



Cause of minor loss	K value or loss expression	Reference
Reentrant Fig. C-17	1.0	3, 4, 5, 8
Submerged pipe exit Fig. C-18	1.0	1, 2, 5, 6, 7, 8, 9
Sudden reduction Fig. C-19	$h_L = \frac{1}{2}\left[1 - \left(\frac{D_2}{D_1}\right)^2\right]\frac{V_2^2}{2g}$	3
Sudden expansion Fig. C-20	$h_L = \frac{(V_1 - V_2)^2}{2g}$ or $h_L = \left(\frac{A_2}{A_1} - 1\right)^2 \frac{V_2^2}{2g}$	1, 2, 3, 5 6, 7, 8, 9

Head Loss in Fittings