Module 3c: Flow in Pipes
Hazen-Williams Equation

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Hazen-Williams Equation

- Based on experimental work
- Used to calculated velocity in a pipe based on the relative roughness and slope of the energy line

\[ V = kCR^{0.63}S^{0.54} \]

Where

- \( V \) = velocity
- \( C \) = factor for relative roughness
- \( R \) = hydraulic radius
- \( S \) = slope of the energy line (head loss divided by pipe length)
- \( k \) = “conversion” factor for unit system
  - \( k = 0.849 \) for units of m/sec
  - \( k = 1.318 \) for units of ft/sec

<table>
<thead>
<tr>
<th>Type of Pipe</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pipes extremely straight and smooth</td>
<td>140</td>
</tr>
<tr>
<td>Pipes very smooth</td>
<td>130</td>
</tr>
<tr>
<td>Smooth wood, smooth masonry</td>
<td>120</td>
</tr>
<tr>
<td>New riveted steel, vitrified clay</td>
<td>110</td>
</tr>
<tr>
<td>Old cast iron, ordinary brick</td>
<td>100</td>
</tr>
<tr>
<td>Old riveted steel</td>
<td>95</td>
</tr>
<tr>
<td>Old iron in bad condition</td>
<td>60 – 80</td>
</tr>
</tbody>
</table>


### Channel Characteristics: Hydraulic Radius and Wetted Perimeter

- **Wetted Perimeter** – surface of pipe or channel where fluid is touching (accounts for areas where friction effects are occurring).
  - Not used to describe any area that is open to the atmosphere since friction contact with the atmosphere is negligible.

### Hazen-Williams Equation

- For circular pipes flowing full, the Hazen-Williams formula can be restated:
  
  For a circular pipe flowing full:
  
  \[ R = \frac{D}{4} \]

  By Continuity:
  
  \[ Q = VA \]
Hazen-Williams Equation

• For circular pipes flowing full, the Hazen-Williams formula can be restated:
  Conversion Factor:

\[
\begin{align*}
    1 \text{ ft}^3/\text{sec} & (7.48 \text{ gal} / \text{ft}^3) = 7.48 \text{ gal} / \text{sec} \\
    7.48 \text{ gal} / \text{sec} (60 \text{sec/min}) & = 448 \text{ gal} / \text{min} \\
    448 \text{ gal} / \text{min} (60 \text{min/hr})(24 \text{hr/day}) & = 646,272 \text{ gal/day} \\
    646,272 \text{ gal/day} & \approx 0.646 \text{ million gal/day} \\
    \text{Conversion Factor} & = 0.646 \text{ MGD/cfs}
\end{align*}
\]

Hazen-Williams Equation

• Substituting into Hazen-Williams:

\[
V[\text{ft}^3/\text{sec}] = 1.318CR0.63S0.54
\]

\[
Q[\text{ft}^3/\text{sec}] = 1.318\frac{\pi}{4}D^2C\left(\frac{D}{4}\right)^{0.63}S^{0.54}
\]

\[
Q = 1.035D^2\left(\frac{D}{4}\right)^{0.63}CS^{0.54}
\]

\[
Q = 0.432CD^{2.63}S^{0.54} \text{ [units of ft}^3/\text{sec and ft]}
\]

Where \(Q = \text{flow} \text{ (MGD)}\)
\(D = \text{diameter} \text{ (ft)}\)

Hazen-Williams Equation

• Substituting into Hazen-Williams:

\[
\begin{align*}
    Q & = 0.432CD^{2.63}S^{0.54} \text{ [units of ft}^3/\text{sec]} \\
    Q & = 0.432CD^{2.63}S^{0.54}(0.646\text{ MGD/cfs}) \\
    Q & = 0.279CD^{2.63}S^{0.54}
\end{align*}
\]

Where \(Q = \text{flow} \text{ (MGD)}\)
\(D = \text{diameter} \text{ (ft)}\)

Hazen-Williams Equation

• For SI units, the Hazen-Williams equation for pipes flowing full:

\[
Q = 0.278CD^{2.63}S^{0.54}
\]

Where \(Q = \text{flow} \text{ (m}^3/\text{sec)}\)
\(D = \text{pipe diameter} \text{ (m)}\)
Hazen-Williams Equation

Example:
- Determine the head loss in a 1000-m pipeline with a diameter of 500 mm that is discharging 0.25 m$^3$/sec. Assume that the Hazen-Williams coefficient for the pipe equals 130.
  
  Given:  
  \[ L = 1000 \text{ m} \]  
  \[ D = 0.5 \text{ m} \]  
  \[ Q = 0.25 \text{ m}^3/\text{sec} \]  
  \[ C = 130 \]

Using the Hazen-Williams equation for flow:

\[ Q = 0.278CD^{2.63}S^{0.54} \]

By definition:

\[ S = h_f/L \]

Substituting:

\[ Q = 0.278CD^{2.63} \left( \frac{h_f}{L} \right)^{0.54} \]

Solving for \( h_f \):

\[ \frac{Q}{0.278CD^{2.63}} = \left( \frac{h_f}{L} \right)^{0.54} \]

\[ \frac{h_f}{L} = \left( \frac{Q}{0.278CD^{2.63}} \right)^{1/0.54} \]

\[ h_f = L \left( \frac{Q}{0.278CD^{2.63}} \right)^{1/0.54} \]

Substituting:

\[ h_f = (1000m) \left( \frac{(0.25m^3/\text{sec})}{0.278(130)(0.5m)^{2.63}} \right)^{1.85} \]

\[ h_f = 2.94m \]
Hazen-Williams Equation

Example:
- A 14-inch diameter schedule 80 pipe has an inside diameter of 12.5 inches (317.5 mm). What is the friction factor $f$ if the pipe is flowing full and the allowable head loss is 3.5 m in a length of 200 m? Use Hazen-Williams equation to calculate velocity.

$$V = kCR^{0.63}S^{0.54}$$

- Assume that the pipe is cast-iron ⇒ $C = 120$
- Calculate the slope of the energy gradient:

$$S = \frac{h_f}{L} = \frac{3.5m}{200m} = 0.0175$$

- Calculate the hydraulic radius, $R$, for pipe flowing full:

$$R = \frac{D}{4} = \frac{0.3175m}{4} = 0.079375m$$

- Substituting into the Hazen-Williams equation:

- For SI units, $k = 0.849$
- Calculate a Reynolds number based on this velocity:

$$Re = \frac{VD}{\nu} = \frac{2.32m/sec(0.3175m)}{1.003x10^{-6}m^2/sec}$$

$$Re = 7.3x10^5$$

Refer to Moody Diagram

Reading from the Moody diagram (since this is based on a previous problem, $\varepsilon/d$ has already been calculated and is 0.000012):

$$f = 0.0125$$
### Hazen-Williams Equation

To correct for C factors not equal to 100 when using the Hazen-Williams nomogram:

- Given flow and diameter, find $S_{100}$ from the nomogram:
  $$S_C = S_{100} \left( \frac{100}{C} \right)^{1.85}$$

- Given flow and slope, find $D_{100}$ from the nomogram:
  $$D_C = D_{100} \left( \frac{100}{C} \right)^{0.38}$$

- Given diameter and slope, find flow from nomogram:
  $$Q_C = Q_{100} \left( \frac{C}{100} \right)$$

### Hazen-Williams Equation

Example (using the nomogram and the conversions):

- A schedule 80 pipe has an inside diameter of 12.5 inches (317.5 mm). What is the velocity if the pipe is flowing full and the allowable head loss is 3.5 m in a length of 200 m? Use Hazen-Williams equation to calculate velocity. Assume $C = 120$.

  - Given: $D = 12.5$ inches
  - $H_L = 3.5$ m
  - $L = 200$ m

  From the nomogram,
  $V = 6.0$ ft/sec (0.3048 m/ft)
  $V = 1.83$ m/sec
Hazen-Williams Equation

- Modifying based on flow conversion (divide both sides by area) and substituting:
  \[ Q_c = Q_{100} \left( \frac{C}{100} \right) \]
  \[ V = \frac{Q}{A} \]
  \[ V_c = \frac{Q_c}{A} = Q_{100} \left( \frac{C}{100} \right) = V_{100} \left( \frac{C}{100} \right) \]
  \[ V_c = V_{100} \left( \frac{C}{100} \right) \]
  \[ V_c = (1.83 \text{ m/sec}) \left( \frac{120}{100} \right) \]
  \[ V_{120} = 2.2 \text{ m/sec} \]

Example:
- Determine the head loss in a 46-cm concrete pipe with an average velocity of 1.0 m/sec and a length of 30 m.
  - Using Hazen-Williams equation:
    \[ V = kCR^{0.63}S^{0.54} \]
  - Since in SI units, \( k = 0.849 \)
    - For a pipe flowing full:
      \[ R = \frac{D}{4} = \frac{0.46 \text{ m}}{4} \]
      \[ R = 0.115 \text{ m} \]

Hazen-Williams Equation

- By definition:
  Slope of energy line = Head Loss/Length of Pipe
  Or
  Head Loss = \( h_L = (\text{Slope})(\text{Pipe Length}) \)
- Let \( C = 130 \) (concrete pipe \( \Rightarrow \) use average of 120 – 140)
- Solve Hazen-Williams for Slope:
  \[ V = kCR^{0.63}S^{0.54} \]
  \[ S^{0.54} = \frac{V}{kCR^{0.63}} \]
  \[ S = \left( \frac{V}{kCR^{0.63}} \right)^{1/0.54} \]

Substituting:
\[ S = \left( \frac{1 \text{ m/sec}}{0.849 \times 130 \times 0.115^{0.63}} \right)^{1.85} \]
\[ S = 0.0015 \]

Head Loss = Slope(Pipe Length)
\[ H_L = 0.0015(30 \text{ m}) = 0.045 \text{ m} \]
Example:

- Find the discharge from a full-flowing cast iron pipe 24 in. in diameter having a slope of 0.004.
- Assume: 5-year old cast iron ⇒ C = 120
- Using Hazen-Williams:
  \[ Q = 0.279CD^{2.63}S^{0.54} \]
- The flow will be given in MGD, but it is needed in \( \text{ft}^3/\text{sec} \).
- From earlier, conversion factor for MGD to cfs is:
  1 \( \text{ft}^3/\text{sec} \) = 0.646 MGD

Modify Hazen-Williams:

- For flow in \( \text{ft}^3/\text{sec} \):
  \[ Q = 0.279CD^{2.63}S^{0.54} \left( \frac{1}{0.646 \text{MGD}} \right) \text{MGD} \]
  \[ Q = 0.432CD^{2.63}S^{0.54} \]
- Substituting:
  \[ Q = 0.432(120) \left( \frac{24 \text{ in}}{12 \text{ in} / \text{ft}} \right)^{2.63} (0.004)^{0.54} \]
  \[ Q = 16.27 \text{ cfs} \]