“Movement of Pollutants in Lakes”

Module 2: Surface Waters, Lecture 3


Simplest pattern of water movement in a lake: wind-driven currents:

Fickian Mixing in Lakes

- A mass of tracer injected into a lake will move by advection with the water currents, but will also spread out into an ever-larger volume of water.
- Given enough time, it will tend to become completely mixed.
- This mixing is primarily due to turbulence, carrying chemicals away from regions of higher concentrations to areas of lower concentrations.
Concentrations for an instantaneous discharge into a two-dimensional body of water (vertically mixed):

\[ C(x, y, t) = \frac{M}{4\pi \sqrt{D_x D_y}} e^{-\left(\frac{(x-V_x t)^2}{4D_x t} + \frac{(y-V_y t)^2}{4D_y t}\right)} \cdot e^{-kt} \]

- \(M\) is the mass of the chemical discharged, per depth of water \([M/L]\)
- \(x\) and \(y\) are the distances from the injection location \([L]\)
- \(t\) is the time lapsed since injection \([T]\)
- \(V_x\) and \(V_y\) are the average velocity in the \(x\) and \(y\) directions \([L/T]\)
- \(D_x\) and \(D_y\) are the Fickian transport coefficients in the \(x\) and \(y\) directions \([L^2/T]\)
- \(K\) is the first-order decay rate constant \([T^{-1}]\)

The depth is the total depth for a vertically well-mixed lake, or the thickness of a layer in a stratified lake.

What would lake temperatures look like in winter (very cold area)?
Thomann and Mueller 1987

\[ W = QeSe + QrSr + Q_t S_t + PAsSp + S_d V \]

- **W**: mass input [M/T]
- **QeSe**: waste effluent discharged to lake
- **QrSr**: mass from main river
- **QtSt**: mass from tributary
- **PAsSp**: mass input from precipitation
- **SdV**: sediment release

**Symbols**:
- **Qe**: effluent discharge
- **Qr**: river flow
- **Qt**: tributary flow
- **Se**: effluent concentration
- **Sr**: river concentration
- **St**: tributary concentration
- **Sp**: rain concentration
- **Sd**: sediment concentration
- **P**: precipitation amount
- **As**: lake surface area
- **V**: lake volume
Concentration increases with time, since the start of the discharge:

\[ K' = Q + kV \]

Change of mass with time = input mass (gain) – mass outflow (loss) – decay (loss)

Assuming a constant \( Q \) and \( k \) over time

Expanding the derivative:

If \( V \) is temporarily constant:

\[ 0 = \frac{dV}{dt} \]

Then:

\[ \frac{dV}{dt} = V \frac{dS}{dt} \]

Concentration decreases from initial conditions (at end of input) by a combination of flushing and decay:

\[ W(t) = V \frac{ds}{dt} + QS + kVs \]

Can simplify if define: \( k' = Q + kV \)

Then:

\[ W(t) = V \frac{ds}{dt} + k's \]

Resulting in:

\[ s = s_0 \exp \left[ - \left( \frac{1}{t_d} + k \right) t \right] \]

Where the lake detention time, \( t_d \) is defined as:

\[ t_d = \frac{V}{Q} \]
Response due to step load = sum of initial condition (from previous load), plus step load effect:

\[
s = \frac{W}{Q+kV} \left(1 - \exp \left[-\left(\frac{Q}{V} + k \right) \right]\right) + s_o \exp \left[-\left(\frac{Q}{V} + k \right) \right]
\]

Total response and transitions can be determined by calculating individual responses and summing the effect.

Response due to varying load:

(a) Step input and subsequent reduction to zero

(b) Step input with reduction to new load level

Example Problem:

A Lake (with initial \( s = 0 \)), receives a load of a slowing reacting pesticide (triallate) of 1080 lb/day for 1.5 years and is then terminated.

1. Determine the equilibrium concentration
2. The maximum concentration
3. The time until a level of 100 µg/L is reached
(1) Determine the equilibrium concentration

\[ \bar{C} = \frac{W}{Q+kV} = \frac{W/Q}{1+kt_d} \]

\[ t_d = \frac{V}{Q} = \frac{3.15 \times 10^6 \text{ ft}^3/\text{sec} \times \frac{\text{day}}{86,400 \text{ sec}} \times \frac{\text{year}}{365 \text{ days}}}{1.0 \text{ year}} = 1.0 \text{ year} \]

\[ \bar{C} = \frac{W/Q}{1+kt_d} = \frac{1080 \text{ lb/day}}{1+\left(0.23 \text{ day}\right)\left(1 \text{ year}\right)} = 0.000102 \text{ lb/ft}^3 \]

\[ \bar{C} = 0.000102 \text{ lb/ft}^3 \times \frac{1 \text{ gal}}{7.48 \text{ gal}} \times \frac{1 \text{ ft}}{3.78 \text{ L}} \times \frac{454,000 \text{ mg}}{1 \text{ lb}} = 1.63 \text{ mg/L} = 1630 \mu \text{g/L} \]

(2) Determine the maximum concentration

(\text{the max. concentration will occur at the end of the discharge time, at } t=1.5 \text{ years})

\[ s = \bar{C} \left[ 1 - \exp \left( -\left(1 + \frac{kt_d}{t_d}\right) \right) \right] \]

\[ s = 1630 \mu \text{g/L} \left[ 1 - \exp \left( -\left(1 + \frac{0.23}{\text{yr}} \times \frac{1.0 \text{ yr}}{1.0 \text{ yr}} \times \frac{1.5 \text{ yr}}{1.0 \text{ yr}} \right) \right) \right] = 1370 \mu \text{g/L} \]

Never reaches the equilibrium concentration before it starts to decrease.

(3) Determine the time until a level of 100 \( \mu \text{g/L} \) is reached

\[ s = s_0 \exp \left( -\left(1 + \frac{Kt_d}{t_d}\right) \left(\frac{t'}{t_d}\right) \right) \]

Where: \( t' = t - 1.5 \text{ years} \)

\[ 100 \mu \text{g/L} = 1370 \mu \text{g/L} \exp \left[ -\left(1 + \frac{0.23}{\text{yr}} \times \frac{1.0 \text{ yr}}{1.0 \text{ yr}} \right) \left(\frac{t'}{t_d}\right) \right] \]

\[ t' = 2.13 \text{ years} \]

Thomann and Mueller 1987